

Definition 1. Let A and B be sets. A *function* from A to B is an assignment of every element in A to a unique element in B . We say that f *maps A into B* .

Let f be a function from A to B . If $a \in A$, the element of B to which a is assigned by f is denoted $f(a)$. Functions satisfy this “defining property”:

$$\text{for every } a \in A \text{ there exists a unique } b \in B \text{ such that } f(a) = b.$$

If f is a function from A to B , this fact is denoted

$$f : A \rightarrow B.$$

- The *domain* of f is A .
- The *codomain* of f is B .
- The *range* of f is $f(A) = \{b \in B \mid b = f(a) \text{ for some } a \in A\}$.

We say that f maps A *onto* B if $f(A) = B$.

Problem 1. For each of the following situations, determine if the assignment is a function from A to B . Explain your reasoning. If “it depends”, say what it depends on. If it is a function, state whether it is “onto”.

- (a) A is the set of grains of sand in the world, B is the set of beaches in the world, assign a grain to a beach.
- (b) A is the set of caged animals in a zoo, B is the set of cages, assign an animal to a cage.
- (c) A is the set of integers, B is the set of integers, a is assigned to b if $b^2 = a$.
- (d) A is the set of men on an island, B is the set of women, a is assigned to b if b is the sister of a .
- (e) A is the set human artifacts on the moon, B is the set of rockets ever to leave earth’s atmosphere, an artifact is assigned to the rocket which delivered it to the moon.
- (f) A is the set of own-goals ever scored, B is the set of goalies, own-goals are assigned to the goalie who scored it.

Problem 2. We would like to define a function $f : \mathbb{Z} \times \mathbb{Z} \rightarrow \mathbb{Q}$ by $(p, q) \mapsto \frac{p}{q}$. Unfortunately, this does not make sense. Fix the problem, so that the resulting function is surjective but not injective.

Problem 3. We would like to define a function $f : \mathbb{Q} \rightarrow \mathbb{Z}$ by $\frac{p}{q} \mapsto pq$. Unfortunately, this is not “well-defined”. Figure out what this means and fix the problem. Is the resulting function injective?